LIBERTY PAPER SET

STD. 10 : Mathematics (Basic) [N-018(E)]

Full Solution

Time : 3 Hours **ASSIGNTMENT PAPER 1**

Section-A

1. (A) One **2.** (B) $b^2 - 4ac = 0$ **3.** (C) 6 **4.** (B) $5\sqrt{2}$ **5.** (B) 1 **6.** (A) 25 **7.** Irrational **8.** $\frac{3}{4}$ **9.** 8 **10.** 1 **11.** 2 **12.** 1 **13.** True **14.** False **15.** False **16.** True **17.** It is a parallel series. **18.** 0 (Zero) **19.** $\frac{5}{6}$ **20.** 6 **21.** (b) πr^2 **22.** (c) $\pi r^2 h$ **23.** (c) $\frac{\pi r\theta}{180}$ **24.** (a) πd

Section-B

25. $x^2 + 7x + 10 = 0$

$\therefore x^2 + 2x + 5x + 10 = 0$

$\therefore x(x + 2) + 5(x + 2) = 0$

$\therefore (x + 2)(x + 5) = 0$

$\therefore x + 2 = 0$ or $x + 5 = 0$

$\therefore x = -2$ or $x = -5$

26. Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$\therefore \alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{4} = \frac{c}{a}$

$\therefore a = 4,\ b = 1$ and $c = 1$

So, one quadratic polynomial which fits the given conditions is $4x^2 + x + 1$. You can check that any other quadratic polynomial that fits these conditions will be of the form $k(4x^2 + x + 1)$, where k is real.

27. $\therefore x^2 - 5x + 2x - 10 = 0$

$\therefore x(x - 5) + 2(x - 5) = 0$

$\therefore (x - 5)(x + 2) = 0$

$\therefore x - 5 = 0$ OR $x + 2 = 0$

$\therefore x = 5$ OR $x = -2$

$\therefore$ The roots of this equation : 5, – 2

28. Here, $a = 2$, $d = 7 - 2 = 5$ and $n = 10$

$a_n = a + (n - 1)d$

$\therefore a_{10} = 2 + (10 - 1)5$

$\therefore a_{10} = 2 + 45$

$\therefore a_{10} = 47$

Therefore, the 10th term of the given AP is 47.

29. Suppose, $S_{1000} = 1 + 2 + 3 + \dots\dots\dots + 1000$

Now, $S_n = \frac{n}{2}(a + a_n)$

$\therefore S_{1000} = \frac{1000}{2}(1 + 1000)$

$\therefore S_{1000} = 500 \times 1001$

$\therefore S_{1000} = 500500$

So, the sum of the first 1000 positive integers is 500500.

30. Let the given points be P(2, 3) & Q(4, 1)

$\therefore$ PQ = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2-4)^2 + (3-1)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Therefore, the distance between the given points is $2\sqrt{2}$.

31. The distance between P (2, –3) and Q (10, y) is 10 units.

$\therefore$ PQ = 10

$\therefore$ $PQ^2 = (10)^2$

$\therefore$ $(2 - 10)^2 + (-3 - y)^2 = 100$

$\therefore$ $64 + 9 + 6y + y^2 - 100 = 0$

$\therefore$ $y^2 + 6y - 27 = 0$

$\therefore$ $y^2 + 9y - 3y - 27 = 0$

$\therefore$ $y(y + 9) - 3(y + 9) = 0$

$\therefore$ $(y + 9)(y - 3) = 0$

$\therefore$ $y + 9 = 0$ OR $y - 3 = 0$

$\therefore$ $y = -9$ OR $y = 3$

Hence, $y = -9$ and 3.

32. *Sin* $\theta = \frac{4}{5}$

In right angled Δ ABC, $\angle B = 90°$ and = $\angle C = \theta$

Sin $\theta = \frac{AB}{AC} = \frac{4}{5}$

$\therefore$ $\frac{AB}{4} = \frac{AC}{5} = K$, K = positive real number

$\therefore$ AB = $4k$, AC = $5k$

According to pythagoras,

$BC^2 = AC^2 - AB^2$

$\therefore$ $BC^2 = (5k)^2 - (4k)^2$

$\therefore$ $BC^2 = 25k^2 - 16k^2$

$\therefore$ $BC^2 = 9k^2$

$\therefore$ BC = $3k$

$\therefore$ *cos* $\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$ and

tan $\theta = \frac{AB}{BC} = \frac{4k}{3k} = \frac{4}{3}$

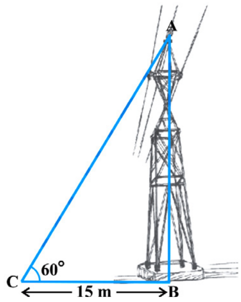
33. ***sin* 60° *cos* 30° + *sin* 30° *cos* 60°**

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

34.



Here, AB represents the tower, CB = 15 is the point from the tower and ∠ACB is the angle of elevation = 60°.

Now, $tan\ 60° = \frac{AB}{BC}$

$\therefore \sqrt{3} = \frac{AB}{15}$

$\therefore AB = 15\sqrt{3}$ m

Hence, the height of the tower is $15\sqrt{3}$ m.

35. Volume of cube = x^3

$\therefore 125 = x^3$

$\therefore (5)^3 = x^3$

$\therefore x = 5$ cm

$\therefore l = 2x = 2(5) = 10$ cm, $b = h = x = 5$ cm

Volume of cuboid $= l \times b \times h$

$= 10 \times 5 \times 5$

$= 250$ cm^3

36.

Cylindrical glass	Hemisphere
$d = 5$ cm.	$d = 5$ cm
$\therefore r = \frac{5}{2}$ cm	$r = \frac{5}{2}$ cm
$\therefore h = 10$ cm	

The Apparent capacity of the glass $= \pi r^2 h$

$= 3.14 \times \frac{5}{2} \times \frac{5}{2} \times 10$

$= 1.57 \times 5 \times 5 \times 5$

$= 196.25$ cm^3

37. Median M $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 60 + \left(\frac{\frac{53}{2} - 22}{7}\right) \times 10$

$= 60 + \frac{(26.5 - 22) \times 10}{7}$

$= 60 + \frac{4.5 \times 10}{7}$

$= 60 + \frac{45}{7}$

$= 60 + 6.43$

$= 66.43$

38. $2x + 3y = 11$ ...(1)

$2x - 4y = -24$ ...(2)

As per equation (1)

$y = \frac{11 - 2x}{3}$ ...(3)

Put value of equation (3) in equation (2)

$2x - 4y = -24$

$\therefore\ 2x - 4\left(\frac{11 - 2x}{3}\right) = -24$

$\therefore\ 6x - 44 + 8x = -72$

$\therefore\ 6x + 8x = -72 + 44$

$\therefore\ 14x = -28$

$\therefore\ x = -2$

Put $x = -2$ in equation (3)

$y = \frac{11 - 2x}{3}$

$\therefore\ y = \frac{11 - 2(-2)}{3}$

$\therefore\ y = \frac{11 + 4}{3}$

$\therefore\ y = 5$

The solution : $x = -2,\ y = 5$

39. By the method of elimination :

$3x - 5y - 4 = 0$ ...(1)

$9x = 2y + 7$

$\therefore\ 9x - 2y - 7 = 0$ ...(2)

Multiply equation (1) by 2 and equation (2) by 5 and subtract

$$\begin{array}{l} 6x \quad - 10y - 8 \quad = 0 \\ 45x - 10y - 35 = 0 \\ - \qquad + \qquad\quad + \\ \hline \end{array}$$

$\therefore\ -39x + 27 = 0$

$\therefore\ -39x = -27$

$\therefore\ x = \frac{27}{39} = \frac{9}{13}$

Put $x = \frac{9}{13}$ in equation (1)

$3x - 5y - 4 = 0$

$\therefore\ 3\left(\frac{9}{13}\right) - 5y - 4 = 0$

$\therefore\ \frac{27}{13} - 5y - 4 = 0$

$\therefore\ \frac{27}{13} - 4 = 5y$

$\therefore\ 65y = 27 - 52$

$\therefore\ 65y = -25$

$\therefore\ y = -\frac{5}{13}$

The solution of the equation : $x = \frac{9}{13},\ y = -\frac{5}{13}$

40. The positive integers that are dvisible by 7 are 7, 14, 21, 28, ...

$a = 7,\ d = 7,\ n = 40$

$\therefore\ S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$\therefore\ S_{40} = \frac{40}{2}\ [2\ (7) + (40 - 1)\ 7]$

$= 20\ [14 + 273]$

$= 20 \times 287$

$\therefore\ 540 = 5740$

41. In a circle, centre is the midpoint of every diameter.

Suppose, A (x, y) and B (1, 4) be the midpoints of the diameter (2, – 3).

Co-ordinates from the midpoint of AB = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\therefore\ (2, -3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$

$\therefore\ 2 = \frac{x+1}{2}$ and $-3 = \frac{y+4}{2}$

$\therefore\ x + 1 = 4$ $\quad y + 4 = -6$

$\therefore\ x = 3$ $\quad y = -10$

Hence, the co-ordinates of A are (3, – 10).

42. A(4, –1) ———— P ———— Q ———— B(–2, –3)

Suppose, A (4, –1) and B (–2, –3) connecting the line segment AB are the trisection points P and Q.

$\therefore\ AP = PQ = QB$

Here, point P divides AB internally in ratio 1 : 2.

$\therefore$ The co-ordinate of point

$$P = \left(\frac{1(-2)+2(4)}{1+2}, \frac{1(-3)+2(-1)}{1+2}\right)$$

$$= \left(2, -\frac{5}{3}\right)$$

Same as, point Q divides AB in ratio 2 : 1.

$\therefore$ The co-ordinate of point

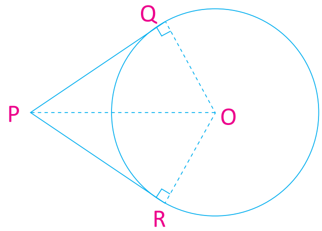
$$Q = \left(\frac{2(-2)+1(4)}{2+1}, \frac{2(-3)+1(-1)}{2+1}\right)$$

$$= \left(0, -\frac{7}{3}\right)$$

43. Given : A circle with centre O, a point P lying outside the circle with two tangents PQ, PR on the circle from P.

To prove : PQ = PR

Figure :



Proof : Join OP, OQ and OR. Then ∠OQP and ∠ORP are right angles because these are angles between the radii and tangents and according to theorem 10.1 they are right angles.

Now, in right triangles OQP and ORP,

OQ = OR (Radii of the same circle)

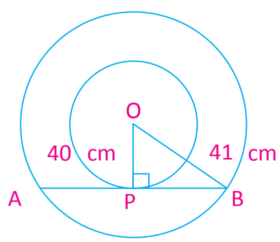
OP = OP (Common)

∠OQP = ∠ORP (Right angle)

Therefore, Δ OQP ≅ Δ ORP (RHS)

This gives, PQ = PR (CPCT)

44.



Here we have,

r_1 = 41 cm

r_2 = 40 cm

Length of chord $= 2\sqrt{r_1^2 - r_2^2}$

$= 2\sqrt{41^2 - 40^2}$

$= 2\sqrt{1681 - 1600}$

$= 2\sqrt{81}$

$= 2\sqrt{81}$

= 2 (9)

= 18 cm.

45. Mode :

Here, maximum class frequency is 23 which belong to class interval 35-45.

∴ l = lower limit of modal class = 35

h = class size = 10

f_1 = frequency of modal class = 23

f_0 = frequency of class prceeding the modal class = 21

f_2 = frequency of class succeeding the modal class = 14

$$\text{Mode, Z} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\therefore \text{Z} = 35 + \left(\frac{23 - 21}{2(23) - 21 - 14}\right) \times 10$$

$$\therefore \text{Z} = 35 + \frac{2 \times 10}{11}$$

∴ Z = 35 + 1.82

∴ Z = 36.82 (Approx)

46. Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180

$\therefore$ Total number of outcomes = 180

(i) Suppose event A is the fallen coin will be a 50 p coin.

$$\therefore P(A) = \frac{\text{Number of 50 p coin}}{\text{Total number of outcomes}}$$

$$\therefore P(A) = \frac{100}{180}$$

$$\therefore P(A) = \frac{5}{9}$$

(ii) Suppose event B is the fallen coin will not be. a ₹ 5 coin, therefore event $\overline{B}$ is the fallen coin will be ₹ 5.

$$\therefore P(\overline{B}) = \frac{\text{Number of ₹ 5 coins}}{\text{Total number of outcomes}}$$

$$\therefore P(\overline{B}) = \frac{10}{180}$$

$$\therefore P(\overline{B}) = \frac{1}{18}$$

Now, $P(B) = 1 - P(\overline{B}) = 1 - \frac{1}{18} = \frac{17}{18}$

$$\therefore P(B) = \frac{17}{18}$$

(iii) Suppose event C is the fallen coin will be ₹ 1 coin.

$$\therefore P(C) = \frac{\text{Number of ₹ 1 coin}}{\text{Total number of outcomes}}$$

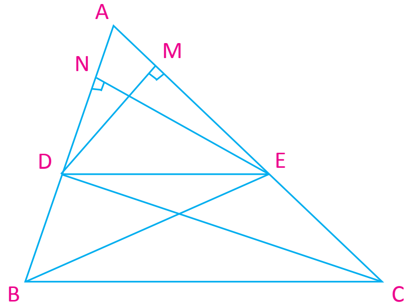
$$\therefore P(C) = \frac{50}{180}$$

$$\therefore P(C) = \frac{5}{18}$$

47. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Proof: Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, $ADE = \frac{1}{2} \times AD \times EN$,

$BDE = \frac{1}{2} \times DB \times EN$,

$ADE = \frac{1}{2} \times AE \times DM$ and

$DEC = \frac{1}{2} \times EC \times DM$.

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

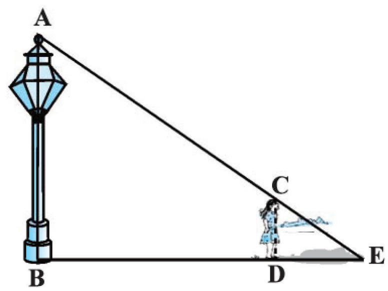
Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq^n. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

48.



Let AB denote the lamp-post, and CD the girl after walking for 4 seconds away from the lamp-post (see Fig.).

From the figure, you can see that DE is the shadow of the girl. Let DE be x metres.

Now, Distance = Speed × Time

∴ BD = 1.2 × 4

∴ BD = 4.8 m

In Δ ABE and Δ CDE

∠B = ∠D (Each is of 90°)

∴ ∠E = ∠E (Same angle)

∴ Δ ABE ~ Δ CDE (AA similarity criterion)

$$\therefore \frac{BE}{DE} = \frac{AB}{CD}$$

$$\therefore \frac{BD + DE}{DE} = \frac{AB}{CD}$$

$$\therefore \frac{4.8 + x}{x} = \frac{3.6}{0.9} \quad (\because 90 \text{ cm} = 0.9 \text{ m})$$

∴ $4.8 + x = 4x$

∴ $3x = 4.8$

∴ $x = 1.6$

So, the shadow of the girl after walking for 4 seconds is 1.6 m long.

49. Suppose, the size of the base = x cm

Hence, the measurement of altitude = $(x - 7)$ cm

According to Pythagoras theorem,

$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$

$\therefore (x)^2 + (x - 7)^2 = (13)^2$

$\therefore x^2 + x^2 - 14x + 49 = 169$

$\therefore 2x^2 - 14x + 49 - 169 = 0$

$\therefore 2x^2 - 14x - 120 = 0$

$\therefore x^2 - 7x - 60 = 0$

$\therefore x^2 - 12x + 5x - 60 = 0$

$\therefore x(x - 12) + 5(x - 12) = 0$

$\therefore (x + 5)(x - 12) = 0$

$\therefore x + 5 = 0$ OR $x - 12 = 0$

$\therefore x = -5$ OR $x = 12$

But the size of the base should not be negative.

The base of the given triangle = 12 cm

The altitude of this triangle will be = 12 – 7 = 5 cm.

50. Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1st, 2nd, 3rd, ..., years will form an AP.

Suppose, denote the number TV sets manufactured in the n^{th} year by a_n

Here, $a_3 = 600$ i.e. $a + 2d = 600$ ...(1)

$a_7 = 700$ i.e. $a + 6d = 700$ ...(2)

Subtract equation (2) by (1),

$(a + 2d) - (a + 6d) = 600 - 700$

$\therefore a + 2d - a - 6d = -100$

$\therefore -4d = -100$

$\therefore d = 25$

Put $d = 25$ in equation (1)

$a + 2d = 600$

$\therefore a + 2(25) = 600$

$\therefore a + 50 = 600$

$\therefore a = 550$

(i) Production of TV sets in the first year is 550.

(ii) Now, $a_{10} = a + 9d = 550 + 9(25) = 550 + 225 = 775$

So, production of TV sets in the 10^{th} year is 775.

(iii) Now, $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$\therefore S_7 = \dfrac{7}{2}[2(550) + (7 - 1)25]$

$\therefore S_7 = \dfrac{7}{2}(1100 + 150)$

$\therefore S_7 = \dfrac{7}{2} \times 1250$

$\therefore S_7 = 4375$

Thus, The total Production of TV sets in first 7 years is 4375.

51. Here we get the information as shown in the table below using $a = 225$ and $h = 50$ to use the deviation method.

Daily expenditure (in ₹)	(f_i)	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100 – 150	4	125	– 2	– 8
150 – 200	5	175	– 1	– 5
200 – 250	12	225 = a	0	0
250 – 300	2	275	1	2
300 – 350	2	325	2	4
Total	$\Sigma f_i = 25$	–	–	$\Sigma f_i u_i = -7$

$$\text{Mean } \bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\therefore \bar{x} = 225 + \frac{-7}{25} \times 50$$

$$\therefore \bar{x} = 225 - 14$$

$$\bar{x} = 211$$

So, mean daliy expenditure on food is ₹ 211.

52.

class	frequency (f_i)	cf
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$

Here, M = 28.5

$n = 60$

Median class = 20 – 30

l = lower limit of median class = 20

n = total frequency = 60

cf = cumulative frequency of class preceding the median class = $5 + x$

f = frequency of median class = 20

h = class size = 10

$$M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore 28.5 = 20 + \left(\frac{\frac{60}{2} - (5 + x)}{20}\right) \times 10$$

$$\therefore 28.5 - 20 = \frac{(30 - 5 - x) \times 10}{20}$$

$$\therefore \frac{8.5 \times 20}{10} = 25 - x$$

$$\therefore 17 = 25 - x$$

$$x = 25 - 17$$

$$x = 8$$

Now, $\Sigma f_i = n = 60$

$\therefore 45 + x + y = 60$

$\therefore 45 + 8 + y = 60$

$\therefore 53 + y = 60$

$\therefore y = 60 - 53$

$\therefore y = 7$

Thus, $x = 8$ and $y = 7$.

53. Here, total number of cards = 52.

$\therefore$ Total number of outcomes = 52

(i) Suppose event C is a red face card. (6)

$$\therefore P(C) = \frac{\text{Number of red face card}}{\text{Total number of outcomes}}$$

$$\therefore P(C) = \frac{6}{52}$$

$$\therefore P(C) = \frac{3}{26}$$

(ii) Suppose event D is jack of hearts (1).

$$\therefore P(D) = \frac{\text{Number of jack of hearts}}{\text{Total number of outcomes}}$$

$$\therefore P(D) = \frac{1}{52}$$

(iii) Suppose event A is an ace of black colour. (2)

$$\therefore P(A) = \frac{\text{Number of an ace of black colour}}{\text{Total number of outcomes}}$$

$$\therefore P(A) = \frac{2}{52}$$

$$\therefore P(A) = \frac{1}{26}$$

(iv) Suppose event B is not an ace. (48)

$$\therefore P(B) = \frac{\text{Number of not an ace}}{\text{Total number of outcomes}}$$

$$\therefore P(B) = \frac{48}{52}$$

$$\therefore P(B) = \frac{12}{13}$$

54. Total number of possible outcomes = 8

(i) Suppose event A points to arrow 8.

$$\therefore P(A) = \frac{\text{Number of results that have 8}}{\text{Total number of outcomes}}$$

$$\therefore P(A) = \frac{1}{8}$$

(ii) Suppose event B is getting to arrow odd numbers. Number of odd number is 4 (1, 3, 5, 7).

$$\therefore P(B) = \frac{\text{Number of odd number}}{\text{Total number of possible outcomes}}$$

$$\therefore P(B) = \frac{4}{8}$$

$$\therefore P(B) = \frac{1}{2}$$

(iii) Suppose event C is getting a number greater than 2 is 6 (3, 4, 5, 6, 7, 8).

$\therefore$ $P(C) = \dfrac{\text{Total number of greater than 2}}{\text{Total number of outcomes}}$

$\therefore$ $P(C) = \dfrac{6}{8}$

$\therefore$ $P(C) = \dfrac{3}{4}$

(iv) Suppose event D points to a number smalles than 9.

Number is smaller than 9 is 8 (1, 2, 3, 4, 5, 6, 7, 8).

$\therefore$ $P(D) = \dfrac{\text{Total number smaller than 9}}{\text{Total number of outcomes}}$

$\therefore$ $P(D) = \dfrac{8}{8}$

$\therefore$ $P(D) = 1$